

**Universal Extra Dimensions and  $b \rightarrow s\gamma$** <sup>1</sup>K. Agashe<sup>2</sup>, N.G. Deshpande<sup>3</sup>, G.-H. Wu<sup>4</sup>*Institute of Theoretical Science**University of Oregon**Eugene OR 97403-5203***Abstract**

We analyze the effect of flat universal extra dimensions (i.e., extra dimensions accessible to all SM fields) on the process  $b \rightarrow s\gamma$ . With one Higgs doublet, the dominant contribution at one-loop is from Kaluza-Klein (KK) states of the charged would-be-Goldstone boson (WGB) and of the top quark. The resulting constraint on the size of the extra dimension is comparable to the constraint from  $T$  parameter. In two-Higgs-doublet model II, the contribution of zero-mode and KK states of physical charged Higgs can cancel the contribution from WGB KK states. Therefore, in this model, there is no constraint on the size of the extra dimensions from the process  $b \rightarrow s\gamma$  and also the constraint on the mass of the charged Higgs from this process is weakened compared to  $4D$ . In two-Higgs-doublet model I, the contribution of the zero-mode and KK states of physical charged Higgs and that of the KK states of WGB are of the same sign. Thus, in this model and for small  $\tan\beta$ , the constraint on the size of the extra dimensions is stronger than in one-Higgs-doublet model and also the constraint on the mass of the charged Higgs is stronger than in  $4D$ .

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<sup>2</sup>email: agashe@neutrino.uoregon.edu

<sup>3</sup>email: desh@oregon.uoregon.edu

<sup>4</sup>email: gwu@electron.uoregon.edu

The motivations for studying theories with *flat* extra dimensions of size  $(\text{TeV})^{-1}$  accessible to (at least some of) the SM fields are varied: SUSY breaking [1], gauge coupling unification [2], generation of fermion mass hierarchies [3] and electroweak symmetry breaking by a composite Higgs doublet [4]. From the  $4D$  point of view, these extra dimensions take the form of Kaluza-Klein (KK) excitations of SM fields with masses  $\sim n/R$ , where  $R$  is a typical size of an extra dimension. In a previous paper [5], we observed that the contribution of these KK states to the process  $b \rightarrow s\gamma$  might give a stringent constraint on  $R^{-1}$ . In this paper, we will analyze in detail the effects of these KK states on the process  $b \rightarrow s\gamma$  both in models with one and two Higgs doublets.

In models with *only* SM gauge fields in the bulk, there are contributions to muon decay, atomic parity violation (APV) etc. from tree-level exchange of KK states of gauge bosons [6, 7]. Then, precision electroweak measurements result in a strong constraint on the size of extra dimensions and, in turn, imply that the effect on the process  $b \rightarrow s\gamma$  is small.

To avoid these constraints, we will focus on models with *universal* extra dimensions, i.e., extra dimensions accessible to *all* the SM fields. In this case, due to conservation of extra dimensional momentum, there are *no* vertices with only one KK state, i.e., coupling of KK state of gauge boson to quarks and leptons always involves (at least one) *KK* mode of quark or lepton. This, in turn, implies that there is no tree-level contribution to weak decays of quarks and leptons, APV  $e^+e^- \rightarrow \mu^+\mu^-$  etc. from exchange of KK states of gauge bosons [8, 9]. However, there is a constraint on  $R^{-1}$  from *one-loop* contribution of KK states of (mainly) the top quark to the  $T$  parameter. For  $m_t \ll R^{-1}$ , this constraint is roughly given by  $\sum_n m_t^2 / (m_t^2 + (n/R)^2) \lesssim 0.5-0.6$  (depending on the neutral Higgs mass) [9]. For the case of one extra dimension, this gives  $R^{-1} \gtrsim 300 \text{ GeV}$ . The KK excitations of quarks appear as heavy stable quarks at hadron colliders and searches by the CDF collaboration also imply  $R^{-1} \gtrsim 300 \text{ GeV}$  for one extra dimension [9].

We begin with an analysis of  $b \rightarrow s\gamma$  for the case of minimal SM with one Higgs doublet in extra dimensions.

## 1 One Higgs doublet

The effective Hamiltonian for  $\Delta S = 1$   $B$  meson decays is

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{j=1}^8 C_j(\mu) \mathcal{O}_j, \quad (1)$$

where the operator relevant for the transition  $b \rightarrow s\gamma$  is

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b \bar{s}_{L\alpha} \sigma^{\mu\nu} b_{R\alpha} F_{\mu\nu}. \quad (2)$$

The coefficient of this operator from  $W - t$  exchange in the SM is

$$C_7^W(m_W) = -\frac{1}{2}A\left(\frac{m_t^2}{m_W^2}\right), \quad (3)$$

where the loop function  $A$  is given by

$$A(x) = x \left[ \frac{\frac{2}{3}x^2 + \frac{5}{12}x - \frac{7}{12}}{(x-1)^3} - \frac{\left(\frac{3}{2}x^2 - x\right) \ln x}{(x-1)^4} \right]. \quad (4)$$

Of course, this includes the contribution from the charged would-be-Goldstone boson (WGB) (i.e., longitudinal  $W$ ).

With extra dimensions, there is a one-loop contribution from KK states of  $W$  (accompanied by KK states of top quark,  $t^{(n)}$ ), but as we show below, this is smaller than that from KK states of charged WGB. In the limit  $m_W \ll R^{-1}$ , the KK states of  $W$  get a mass  $\sim n/R$  by “eating” the field corresponding to extra polarization of  $W$  in higher dimensions<sup>5</sup> – this field is a scalar from the  $4D$  point of view. Thus, the coupling of *all* components of  $W^{(n)}$  to fermions is  $g$ , unlike the case of the zero-mode, where the coupling of *longitudinal*  $W$  to fermions is given by the Yukawa coupling of Higgs to fermions. Therefore, the contribution of  $W^{(n)}$  to the coefficient of the dimension-5 operator  $\bar{s}\sigma_{\mu\nu}bF^{\mu\nu}$  is  $\sim e m_b g^2 / (16\pi^2) m_t^2 \sum_n 1/(n/R)^4$ , where the factor  $m_t^2$  reflects GIM cancellation. In terms of the operator  $\mathcal{O}_7$ , the contribution of each KK state of  $W$  to  $C_7$  is  $\sim m_t^2 m_W^2 / (n/R)^4$ .

From the above discussion, it is clear that the *KK* states of charged would-be-Goldstone boson (denoted by  $\text{WGB}^{(n)}$ ) are physical (unlike the *zero-mode*). The loop contribution of  $\text{WGB}^{(n)}$  with mass  $n/R$  (and  $t^{(n)}$  with mass  $\sqrt{m_t^2 + (n/R)^2}$ ) is of the same form as that of physical charged Higgs in 2 Higgs doublet models [10] with the appropriate modification of masses and couplings of virtual particles in the loop integral

$$C_7^{\text{WGB}^{(n)}}(R^{-1}) \approx \frac{m_t^2}{m_t^2 + (n/R)^2} \left[ B\left(\frac{m_t^2 + (n/R)^2}{(n/R)^2}\right) - \frac{1}{6}A\left(\frac{m_t^2 + (n/R)^2}{(n/R)^2}\right) \right]. \quad (5)$$

Here, the factor  $m_t^2 / (m_t^2 + (n/R)^2)$  accounts for (a) the coupling of  $\text{WGB}^{(n)}$  to  $t^{(n)}$  which is  $\lambda_t \sim m_t/v$ , i.e., the same as that of  $\text{WGB}^{(0)}$  (longitudinal  $W$ ), and (b) the fact that this contribution decouples in the limit of large KK mass – the functions  $A$  and  $B$  (see below) in the above expression approach a constant as  $n/R$  becomes large.

The loop function  $B$  is given by [10]

$$B(y) = \frac{y}{2} \left[ \frac{\frac{5}{6}y - \frac{1}{2}}{(y-1)^2} - \frac{\left(y - \frac{2}{3}\right) \ln y}{(y-1)^3} \right]. \quad (6)$$

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<sup>5</sup>This can be seen from KK decomposition of fields in the  $5D$  gauge kinetic term (see, for example, appendix C of 2nd reference in [2]).

It is clear that the ratio of the contribution of  $W^{(n)}$  and that of  $WGB^{(n)}$  is  $\sim (m_W R/n)^2 \lesssim O(1/10)$  since  $R^{-1} \gtrsim 300$  GeV (due to constraints from the  $T$  parameter and searches for heavy quarks). In what follows, we will neglect the  $W^{(n)}$  contribution.

At NLO, the coefficient of the operator at the scale  $\mu \sim m_b$  is given by [11]

$$C_7(m_b) \approx 0.698 C_7(m_W) - 0.156 C_2(m_W) + 0.086 C_8(m_W). \quad (7)$$

Here,  $C_2$  is the coefficient of the operator  $\mathcal{O}_2 = (\bar{c}_{L\alpha} \gamma^\mu b_{L\alpha}) (\bar{s}_{L\beta} \gamma_\mu c_{L\beta})$  and is approximately same as in the SM (i.e., 1) since the KK states of  $W$  do not contribute to it at tree-level.  $C_8$  is the coefficient of the chromomagnetic operator  $\mathcal{O}_8 = g_s/(16\pi^2) m_b \bar{s}_{L\alpha} \sigma^{\mu\nu} T_{\alpha\beta}^a b_{R\beta} G_{\mu\nu}^a$ . In the SM,  $C_8(m_W) \approx -0.097$  [11] due to the contribution of  $W-t$  loop (using  $m_t \approx 174$  GeV). The coefficient of this operator also gets a loop contribution from KK states which is of the same order as the contribution to  $C_7$ . Since the coefficient of  $C_8$  in Eq. (7) is small, we neglect the contribution of KK states to  $C_8$ . The coefficient of  $\mathcal{O}_7$  at the scale  $m_W$  is given by the sum of the contributions of  $W^{(0)}$  (Eq. (3)) and that of  $WGB^{(n)}$  (Eq. (5)) summed over  $n$ <sup>6</sup>. Since  $C_7^W(m_W) < 0$  and  $C_7^{WGB^{(n)}}(R^{-1}) > 0$ , we see that contribution from  $WGB^{(n)}$  interferes destructively with the  $W$  contribution.

The SM prediction for  $\Gamma(b \rightarrow s\gamma)/\Gamma(b \rightarrow cl\nu)$  has an uncertainty of about 10% and the experimental error is about 15% (both are  $1\sigma$  errors) [12]. The central values of theory and experiment agree to within  $1/2 \sigma$ . The semileptonic decay is not affected by the KK states (at tree-level). Combining theory and experiment  $2\sigma$  errors in quadrature, this means that the 95% CL constraint on the contribution of KK states is that it should not modify the SM prediction for  $\Gamma(b \rightarrow s\gamma)$  by more than 36%. Since  $\Gamma(b \rightarrow s\gamma) \propto [C_7(m_b)]^2$ , the constraint is  $\left| [C_7^{\text{total}}(m_b)]^2 / [C_7^{\text{SM}}(m_b)]^2 - 1 \right| \lesssim 36\%$ . Using  $m_t \approx 174$  GeV, we get  $A \approx 0.39$  in Eq. (3) and  $C_7^{\text{SM}}(m_b) \approx -0.3$ <sup>7</sup> from Eq. (7). Assuming  $m_t \ll R^{-1}$ , we get  $B \approx 0.19$  and  $A \approx 0.21$  in Eq. (5). Then, using Eq. (7) and the above criterion, we get the constraint

$$\sum_n m_t^2 / (m_t^2 + (n/R)^2) \lesssim 0.5 \quad (8)$$

which is comparable to that from the  $T$  parameter. For one extra dimension, performing the sum over KK states with the exact expressions for  $A$  and  $B$  in Eq. (5), the constraint is  $R^{-1} \gtrsim 280$  GeV<sup>8</sup>.

Next, we consider models with two Higgs doublets.

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<sup>6</sup>We neglect the RG scaling of  $\mathcal{O}_7$  between  $R^{-1}$  and  $m_W$ .

<sup>7</sup>The NNLO corrections for the SM prediction of the rate for  $b \rightarrow s\gamma$  are also known and are about a few percent.

<sup>8</sup>We assume that the extra dimension denoted by  $y$  is compactified on a circle of radius  $R$ . The various fields are chosen to be either even or odd under the  $Z_2$  symmetry,  $y \rightarrow -y$  as in [9]. Thus, the summation is over positive integers  $n$ .

## 2 Two-Higgs-doublet model II

In this case, contribution from zero-mode physical charged Higgs interferes constructively with the  $W$  contribution [10]:

$$C_{7,II}^{H^+(0)}(m_W) \approx -B \left( \frac{m_t^2}{m_H^2} \right) - \frac{1}{6} \cot^2 \beta A \left( \frac{m_t^2}{m_H^2} \right), \quad (9)$$

where  $\tan \beta$  is the ratio of vev's of the two Higgs doublets. In  $4D$ , this contribution gives a strong constraint on charged Higgs mass,  $m_H \gtrsim 500$  GeV.

The combined effect from KK states of physical charged Higgs and WGB is:

$$C_{7,II}^{(WGB^{(n)}+H^{+(n)})}(R^{-1}) \approx \frac{m_t^2}{m_t^2 + (n/R)^2} \left[ B \left( \frac{m_t^2 + (n/R)^2}{(n/R)^2} \right) - \frac{1}{6} A \left( \frac{m_t^2 + (n/R)^2}{(n/R)^2} \right) - \right. \\ \left. B \left( \frac{m_t^2 + (n/R)^2}{m_H^2 + (n/R)^2} \right) - \frac{1}{6} \cot^2 \beta A \left( \frac{m_t^2 + (n/R)^2}{m_H^2 + (n/R)^2} \right) \right], \quad (10)$$

where the first line is from KK states of WGB (Eq. (5)) and the second line is from KK states of physical charged Higgs (KK analog of Eq. (9)).

Assuming  $m_H \sim O(R^{-1})$  or larger, the combined effect of KK states is typically destructive with respect to the  $W$  contribution. This is because  $B \left( \frac{m_t^2 + (n/R)^2}{m_H^2 + (n/R)^2} \right) < B \left( \frac{m_t^2 + (n/R)^2}{(n/R)^2} \right)$  and the  $A$  contribution is small such that  $C_{7,II}^{(WGB^{(n)}+H^{+(n)})} > 0$ <sup>9</sup>. Thus, the contribution of zero-mode physical charged Higgs can cancel that of KK states so that there is no constraint on  $R^{-1}$ . Also, this implies that the constraint on  $m_H$  is weakened in the presence of extra dimensions of size  $O(m_H^{-1})$  or larger.

This can be seen in Fig. 1 which shows the deviation in the rate for  $b \rightarrow s\gamma$  from the SM prediction for the case of one extra dimension. From Fig. 1a, we see that even for  $R^{-1}$  as small as 200 GeV<sup>10</sup>, the 95% CL constraint from  $b \rightarrow s\gamma$  is satisfied for a particular range of  $m_H$ . Of course, for  $m_H \gtrsim 1$  TeV, the effect of physical charged Higgs (both zero-mode and KK states) becomes negligible so that we obtain the lower limit on  $R^{-1}$  of about 300 GeV as in the one Higgs doublet case. As seen from Fig. 1b, the 95% CL lower limit from  $b \rightarrow s\gamma$  on  $m_H$  is about 500 – 550 GeV (depending on  $\tan \beta$ ) in  $4D$ . We see that in  $5D$ , the 95% CL lower limit on  $m_H$  is reduced by about 40 GeV for  $R^{-1} \sim 300$  GeV and the  $1\sigma$  limit on  $m_H$  is reduced by about 200 GeV.

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<sup>9</sup>In the limit  $m_H \ll R^{-1}$ , the  $B$ 's cancel in Eq. (10) so that the combined effect of KK states is constructive (and small) due to the  $A$ 's.

<sup>10</sup>Of course, such a small  $R^{-1}$  might be ruled out due to constraints from  $T$  parameter and heavy quark searches.

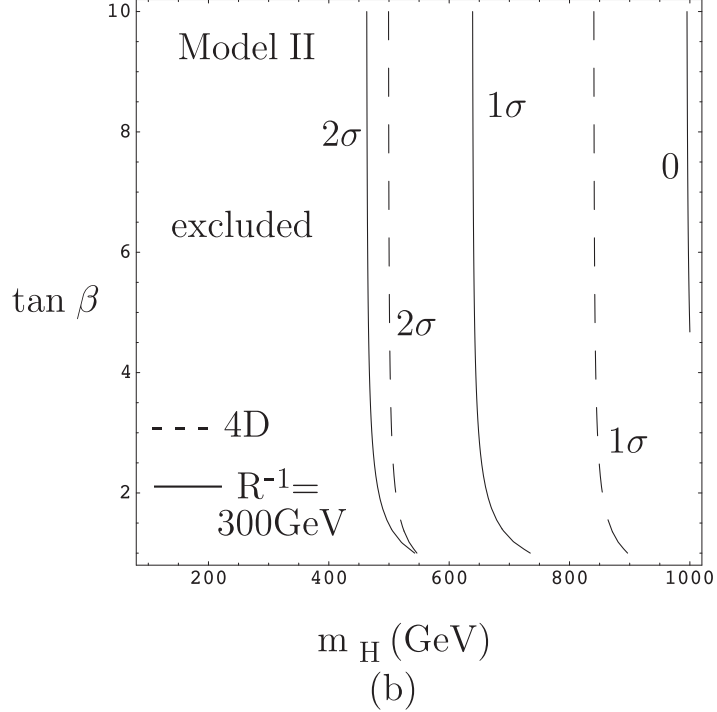
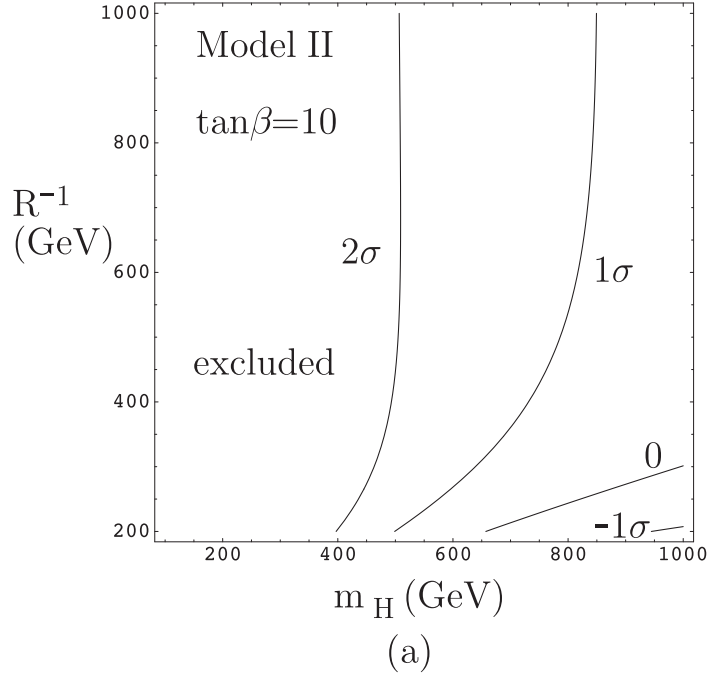


Figure 1: The deviation of the rate of  $b \rightarrow s\gamma$  from the SM prediction in model II as a function of size of one extra dimension ( $R^{-1}$ ) and charged Higgs mass ( $m_H$ ) for  $\tan \beta = 10$  (figure (a)) and as a function of  $\tan \beta$  and  $m_H$  for  $R^{-1} = 300$  GeV (figure (b)). In figure (b), the dashed lines are the result in 4D. The  $1\sigma$  deviation corresponds to 18%.

### 3 Two-Higgs-doublet model I

In this case, the contribution from zero-mode physical charged Higgs is destructive with respect to the  $W$  contribution [10]:

$$C_{7,I}^{H^{+ (0)}}(m_W) \approx \cot^2 \beta \left[ B \left( \frac{m_t^2}{m_H^2} \right) - \frac{1}{6} A \left( \frac{m_t^2}{m_H^2} \right) \right]. \quad (11)$$

This contribution is negligible for large  $\tan \beta$  and hence there is no constraint on  $m_H$  from  $b \rightarrow s\gamma$  in  $4D$ . Of course, for small  $\tan \beta$ , this process does give a lower limit on  $m_H$ : for  $\tan \beta = 1$ , the limit is about 350 GeV.

The contribution from KK states is also destructive with respect to the  $W$  contribution:

$$C_{7,I}^{(\text{WGB}^{(n)} + H^{+ (n)})}(R^{-1}) \approx \frac{m_t^2}{m_t^2 + (n/R)^2} \left[ B \left( \frac{m_t^2 + (n/R)^2}{(n/R)^2} \right) - \frac{1}{6} A \left( \frac{m_t^2 + (n/R)^2}{(n/R)^2} \right) + \cot^2 \beta \left( B \left( \frac{m_t^2 + (n/R)^2}{m_H^2 + (n/R)^2} \right) - \frac{1}{6} A \left( \frac{m_t^2 + (n/R)^2}{m_H^2 + (n/R)^2} \right) \right) \right], \quad (12)$$

where the first line is from KK states of WGB (Eq. (5)) and the second line is from KK states of physical charged Higgs (KK analog of Eq. (11)).

Thus, for small  $\tan \beta$ , the constraint on  $R^{-1}$  is stronger than with one Higgs doublet and also the lower limit on  $m_H$  is larger with extra dimensions.

In Fig. 2, we show the deviation from the SM prediction for the rate of  $b \rightarrow s\gamma$  for the case of one extra dimension. From Fig. 2a, we see that for  $\tan \beta = 2$  and  $m_H = 100$  GeV, the lower limit on  $R^{-1}$  is about 550 GeV (as compared to about 300 GeV in the one Higgs doublet case). Of course, for  $m_H \gtrsim 1$  TeV, the contribution of physical charged Higgs (both zero-mode and KK states) is negligible and then the lower limit on  $R^{-1}$  is the same as in the one Higgs doublet case. From Fig. 2b, we see that for  $\tan \beta = 1$  and  $R^{-1} = 300$  GeV, the limit on  $m_H$  increases from about 350 GeV in  $4D$  to a value much larger than 1 TeV. As another example, for  $\tan \beta = 4$ , there is no constraint on  $m_H$  in  $4D$ , whereas for one extra dimension of size  $(300 \text{ GeV})^{-1}$  there is a lower limit on  $m_H$  of about 400 GeV. However, as in  $4D$ , the effect of physical charged Higgs (both zero-mode and KK states) “decouples” as  $\tan \beta$  becomes larger and then we recover the one Higgs doublet result for  $b \rightarrow s\gamma$ .

### 4 Summary

In this paper, we have studied the effect of universal extra dimensions on the process  $b \rightarrow s\gamma$ . In the one Higgs doublet case, we showed that the contribution of KK states of charged would-be-Goldstone boson (WGB) gives a constraint on the size of the extra dimensions which is comparable to that from the  $T$  parameter. In two-Higgs-doublet model II, the contribution of

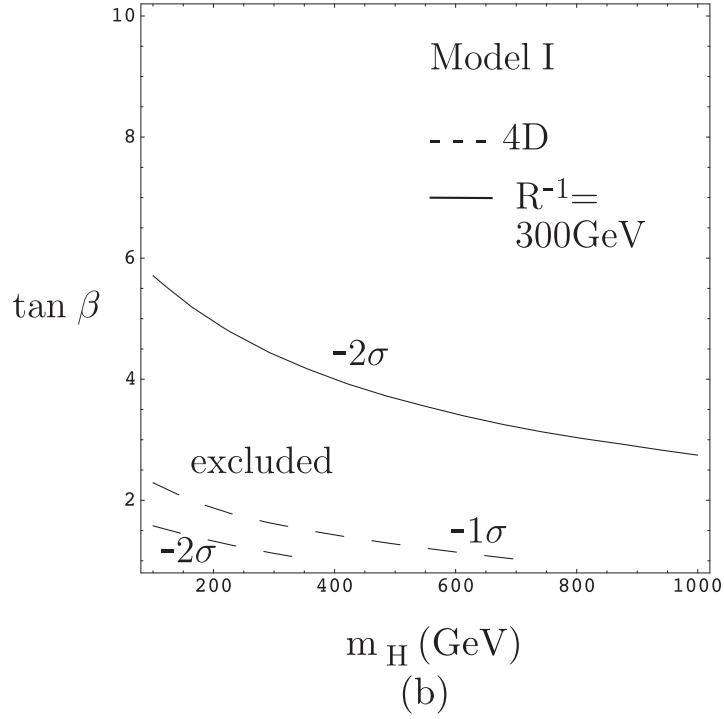
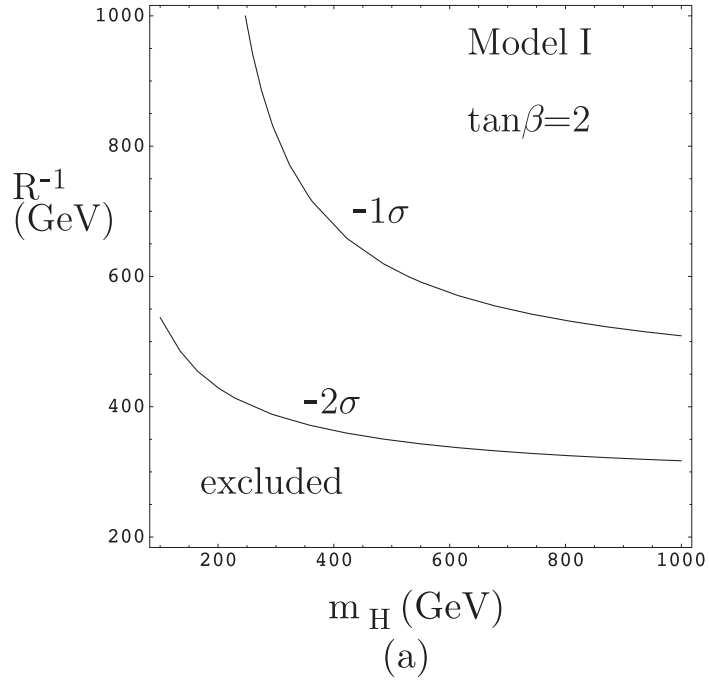


Figure 2: The deviation of the rate of  $b \rightarrow s\gamma$  from the SM prediction in model I as a function of size of one extra dimension ( $R^{-1}$ ) and charged Higgs mass ( $m_H$ ) for  $\tan \beta = 2$  (figure (a)) and as a function of  $\tan \beta$  and  $m_H$  for  $R^{-1} = 300$  GeV (figure (b)). In figure (b), the dashed lines are the result in 4D. The  $1\sigma$  deviation corresponds to 18%.



physical charged Higgs (and its KK states) tends to cancel the contribution of KK states of WGB so that there is no constraint on the size of the extra dimensions and also the lower limit on the charged Higgs mass is relaxed relative to  $4D$ . In two-Higgs-doublet model I, the contribution of physical charged Higgs (and its KK states) adds to the contribution of KK states of WGB. Therefore, for small  $\tan\beta$ , the constraint on the size of extra dimensions becomes stronger than in the one-Higgs-doublet model and also the lower limit on charged Higgs mass is larger than in  $4D$ .

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